WESLEY COLLEGE

By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER ONE 2019
TEST 2: Functions

Name: ________________________

Friday 5th April

Time: 45 minutes

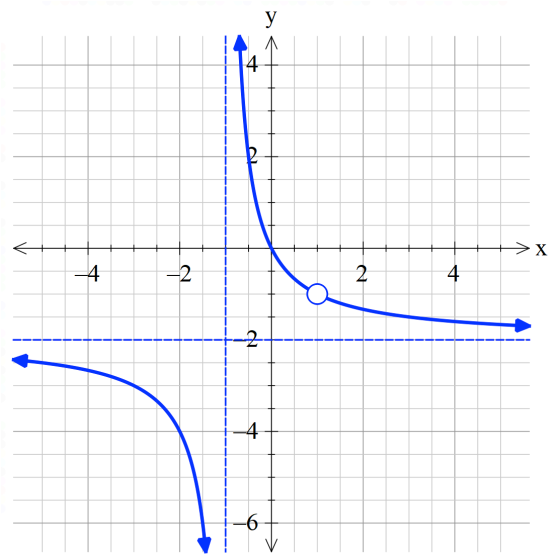
Total marks: $\frac{\quad}{19} + \frac{\quad}{26} = \frac{\quad}{45}$

Calculator free section – maximum 19 minutes

1. [4 marks – 1 each]

   This graph is of a function $y = f(x)$ which has a point discontinuity at $(1, -1)$, with asymptotes and intercept as shown.

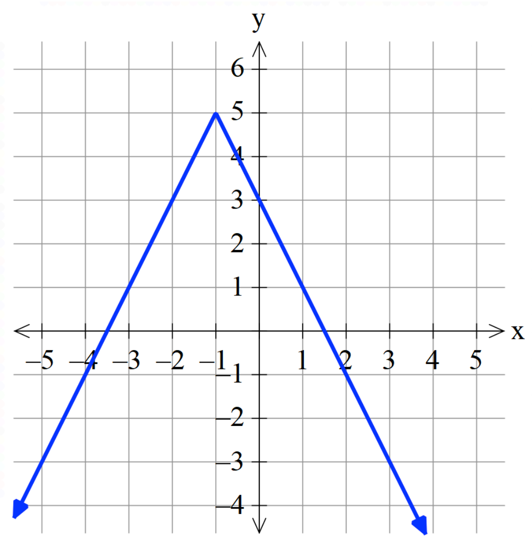
   If $f(x) = \dfrac{a(x-b)(x-c)}{(x-c)(x-d)}$, evaluate a, b, c and d.



2. [5 marks – 3 and 2]

   This graph can be represented by $y = f(x) = a + b|x + c|$

   (a) Evaluate a, b and c



   (b) Add $y = |2x - 3|$ to the graph and determine the values of x for which $|2x - 3| = f(x)$

3. [10 marks – 1, 1, 1, 2, 2, 1 and 2]

$f(x) = \sqrt{x+3}$ and $g(x) = 4 - x^2$

Determine:

(a) the domain of $f(x)$

(b) the range of $g(x)$

(c) $f \circ g(-1)$

(d) x if $f \circ f(x) = 2$

(e) the domain of $g \circ f(x)$

(f) the range of $g \circ f(x)$

(g) which, if any, of these functions has a properly defined inverse. Justify your choice.

Year 12 Specialist Test 2: Functions

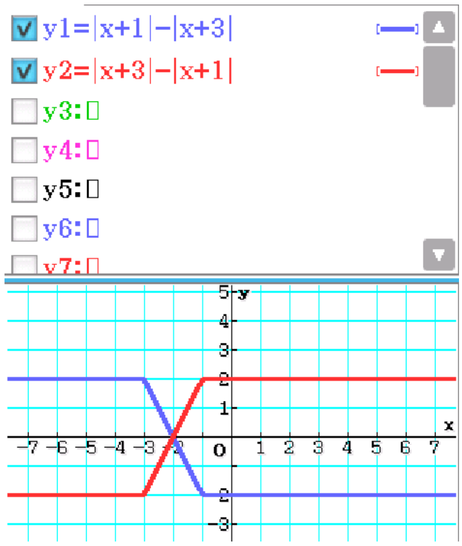
Name: ______________________

Time: 26 minutes   26 marks

Calculator assumed section

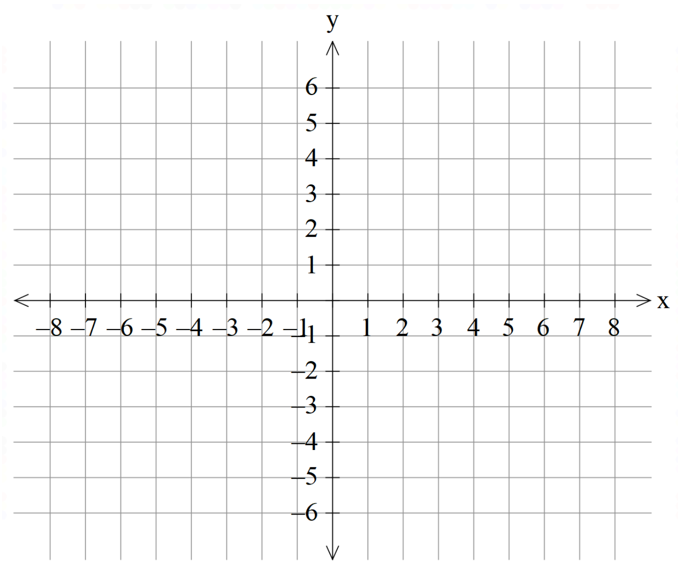
4. [7 marks –2, 2 and 3]

This screenshot shows the graphs of $y_1 = |x+1| - |x+3|$ and $y_2 = |x+3| - |x+1|$

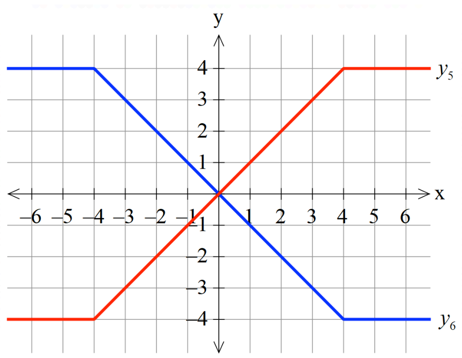


(a) Write a piecewise (algebraic) definition of y_1

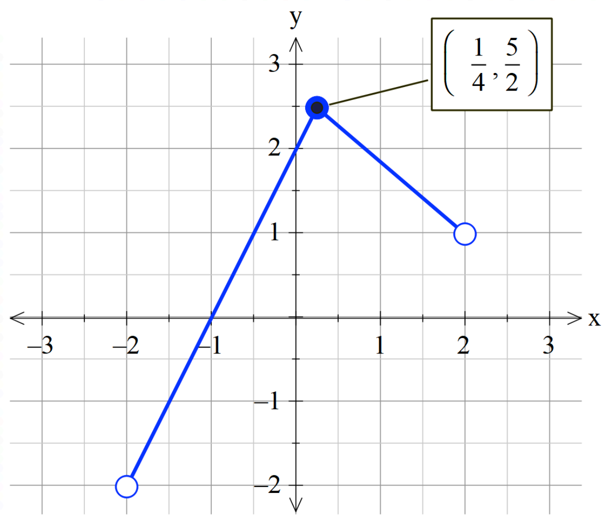
(b) Graph $y_3 = |2x+1| - |2x-5|$ and $y_4 = |2x-5| - |2x+1|$ on these axes:

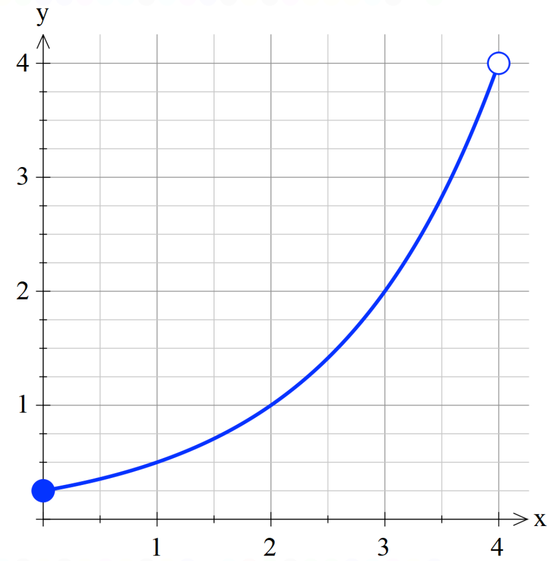


(c) Use differences of absolute values to write the equations of y_5 and y_6 for:



5. [13 marks – 3, 2, 2, 1, 2 and 3]





Graphs of $y = f(x)$ and $y = g(x) = 2^{x-2}$ are given over the restricted domains shown.

Determine:

(a) the domain and range of $f \circ g(x)$

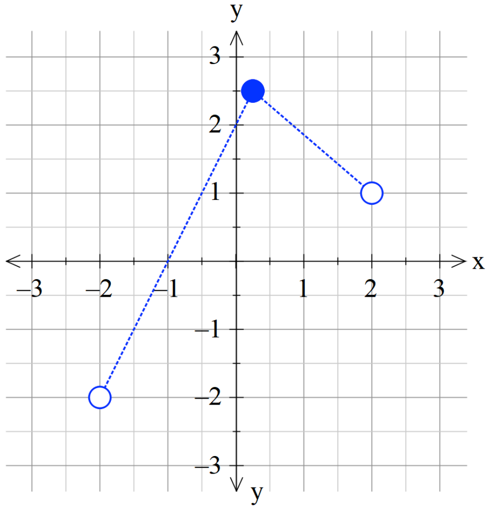
(b) the domain and range of $g \circ f(x)$

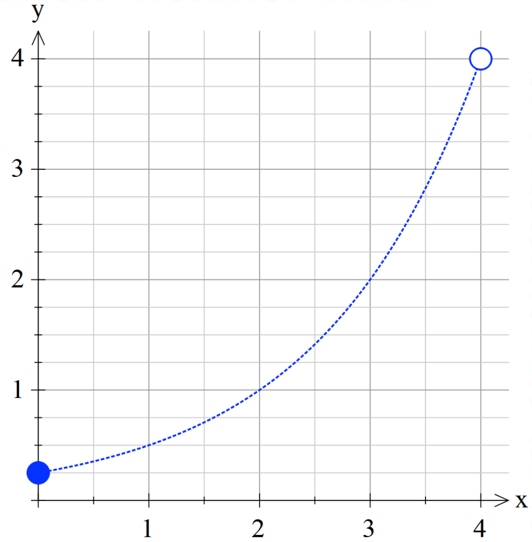
On these separate axes, sketch:

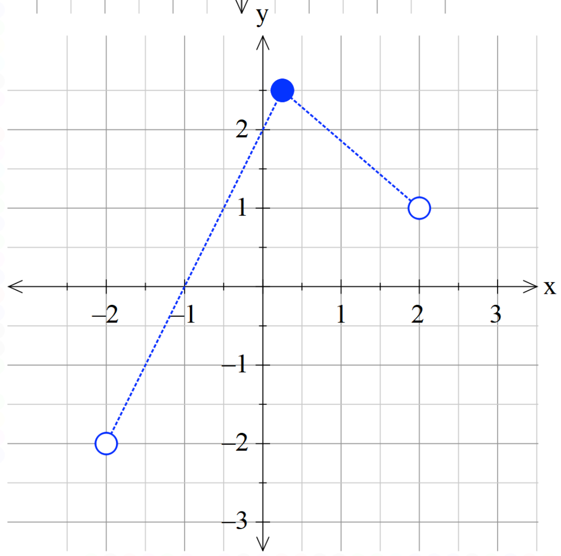
(c) $y = \dfrac{1}{f(x)}$

(d) $y = g^{-1}(x)$

(e) $y = -f(|x|)$







Calculate:

(f) a simplified algebraic expression for $g^{-1}(x)$ over a specified domain.

6. [6 marks –2, 1 and 3]

(a) For $f(x)=\dfrac{2x+3}{3x+2}$, show that $f\left(\dfrac{1}{x}\right)=\dfrac{1}{f(x)}$

(b) Give a further example of a function with $f\left(\dfrac{1}{x}\right)=\dfrac{1}{f(x)}$

(c) Is $f\left(\dfrac{1}{x}\right)=\dfrac{1}{f(x)}$ universally true? Explain and/or justify your conclusion, with reference to at least two further functions.